FLOW OF A MAGNETIZABLE FLUID AT TEMPERATURES CLOSE TO THE CURIE POINT

V. I. Vishnyakov, I. F. Sulmanov, and I. A. Fedomov

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Magnetizable fluids are currently widely used in heat and power engineering. Such fluids can be successfully used as coolants in case where ordinary fluids require additional devices and energy consumption, for example under conditions of weightlessness. The possibility of using magnetizable fluids in electrical generators [1] opens up new prospects for the direct conversion of thermal energy into electrical energy. The effectiveness of heat exchange devices and power installations is determined to a significant degree by the working temperature of the magnetizable fluid. The largest energy effects in many cases are obtained when the temperature is close to the Curie point [1]. However close to the Curie temperature the fluid partially loses its magnetic properties and this will naturally affect its flow properties. In addition, the magnetocaloric effect will also affect the flow and this reaches a maximum at the Curie point [2]. Although this is a small effect, the resulting redistribution of temperature over the width of the channel in the steady-state is significant. We consider the nonisothermal stationary flow of an incompressible, nonconducting, magnetizable fluid in a plane channel when the temperature is close to the Curie point. We assume that the fluid is magnetized to saturation by a strong external magnetic field H directed perpendicular to the channel. Then the magnetic force in the equation of motion can be written in the form $\mu_0 M \nabla H$ [3], where $M = M(T)$ is the bulk magnetization, and μ_0 is the magnetic constant. We also assume that constant heat sources Q are uniformly distributed over the entire volume of the fluid.

Consider a plane channel of width 2D in which a magnetizable fluid moves under the action of an external magnetic field gradient G directed along the channel. The temperature T_0 of the channel walls is held constant at a value close to the Curie point. Then the equations of motion and heat conduction for the magnetizable fluid can be written in the form [3, 4]

$$\eta d^2U/dY^2 + \mu_0 MG = 0; \tag{1}$$

$$\lambda d^2T/dY^2 + \mu_0 T_0 \Lambda GU + Q = 0, \tag{2}$$

where U is the x-component of the velocity vector, T is the temperature, $G = \partial H/\partial X \equiv \mathrm{const}$; $\Lambda = -(\partial M/\partial T)_{\rho, H}$ is the pyromagnetic coefficient, ρ is the density, λ and η are the thermal conductivity and dynamical viscosity, respectively. The Y-axis is directed perpendicular to the walls of the channel, while the X axis goes down its axis. Because the change in temperature across a section of the channel will be small in comparison to the temperature T_0 itself, in the second term of (2) (the magnetocaloric effect) we can put $T = T_0$. The boundary conditions are obvious physically:

$$U = 0,\ T = T_0 \text{ for } (Y = \pm D). \tag{3}$$

The solution of the boundary value problem defined by (1), (2), and (3) depends on the form of the function $M = M(T)$. It is usual in the solution of this kind of problem to approximate M(T) by segments such that either M is constant or a linear function of temperature [3-5] over the temperature range under consideration. At temperatures close to the Curie point this method breaks down because here M(T) is extremely nonlinear. Also for large external magnetic fields the magnetization asymptotically goes to zero with increasing temperature [2]. Hence it is necessary to use a realistic M(T) dependence and this leads to difficulties in finding the solution due to the mathematical complexity of the problem and the necessity to take into account the properties of actual magnetizable fluids. However it is possible to significantly simplify the problem in some cases by approximating the real M(T) curve by a simpler one. In our treatment, the $M = M(T)$ curve is approximated by two

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straight lines. In Fig. 1 the actual M(T) curve is shown as a dashed line and the approximate one as the dolid lines. Below we will discuss when this approximation is valid.

Thus we assume that $M = M(T)$ can be described by the function $M = \Lambda(T_C - T)$, where T_C is the Curie temperature. We construct separate solutions for temperatures below the Curie point where $M > 0$ and for temperatures equal or above the Curie point where $M = 0$. We choose the following scales of measurement: the length D, temperature $T_C - T_0$, and velocity $(T_C - T_0)\sqrt{\lambda/\eta T_0}$. Then the original equations (1) and (2) can be written in dimensionless form for each of the temperature regions:

$$d^2u_1/dy^2 + 2k^2(1-\theta_1) = 0,\ d^2\theta_1/dy^2 + 2k^2u_1 + q = 0 \text{ for } (T < T_C); \quad (4)$$

$$d^2u_2/dy^2 = 0,\ d^2\theta_2/dy^2 + q = 0 \text{ for } (T \geqslant T_C), \quad (5)$$

where $\theta_{1,2} = (T - T_0)/(T_C - T_0)$; $q = D^2Q/(\lambda(T_C - T_0))$; $2k^2 = \mu_0\Lambda G D^2\sqrt{T_0/\eta\lambda}$, $k > 0$. Boundary condition (3) takes the form

$$u_1 = \theta_1 = 0 \text{ for } (y = \pm 1). \quad (6)$$

Solving the system of equations (4) with boundary conditions (6) we find

$$u_1 = \frac{(2k^2 \operatorname{sh} k \sin k + q \operatorname{ch} k \cos k) \operatorname{ch} ky \cos ky - (2k^2 \operatorname{ch} k \cos k - q \operatorname{sh} k \sin k) \operatorname{sh} ky \sin ky}{2k^2(\operatorname{sh}^2 k + \cos^2 k)} - \frac{q}{2k^2},$$

$$\theta_1 = 1 - \frac{(2k^2 \operatorname{sh} k \sin k + q \operatorname{ch} k \cos k) \operatorname{sh} ky \sin ky + (2k^2 \operatorname{ch} k \cos k - q \operatorname{sh} k \sin k) \operatorname{ch} ky \cos ky}{2k^2(\operatorname{sh}^2 k + \cos^2 k)}. \quad (7)$$

The solution (7) is valid for $q < 2$ and $k < k_0$ where k_0 $(0 < k_0 \leqslant \pi/2)$ is found from the solution of the transcendental equation $2k_0^2 \cosh k_0 \cos k_0 = q \sinh k_0 \sin k_0$. When $k = k_0$ the temperature in the center of the channel reaches the Curie point. Because of the magnetocaloric effect, further increase in k leads to an extension of the Curie point to fluid layers lying further from the center of the channel. Thus a certain region of the channel $-y_0 < y < y_0$ is heated above the Curie temperature.

For $q < 2$, $k \geqslant k_0$ and for $q \geqslant 2$, because of the symmetry of the problem relative to the center of the channel, it is convenient to look for the solution of (4) and (5) in the region $0 \leqslant y \leqslant 1$. The relevant boundary conditions are $du_2/dy = d\theta_2/dy = 0$ at $y = 0$ and $u_1 = \theta_1 = 0$ at $y = 1$. In addition at $y = y_0$ we must have

$$u_1 = u_2,\ \theta_1 = \theta_2 = 1,\ du_1/dy = du_2/dy,\ d\theta_1/dy = d\theta_2/dy.$$

We then obtain $y_0 = 1 - a/k$,

$$u_2 = \frac{1 - \operatorname{ch} a \cos a + (k - a)(\operatorname{ch} a \sin a - \operatorname{sh} a \cos a)}{\operatorname{sh} a \sin a + (k - a)(\operatorname{sh} a \operatorname{ch} a + \sin a \cos a)},$$

$$\theta_2 = 1 + \frac{q}{2}(y_0^2 - y^2),$$

where a $(0 < a < \pi/2,\ a < k)$ which is found from solving the transcendental equation $2k^2 \cosh a \cos a = q[\sinh a \sin a + (k - a)(\sinh a + \sin a \cos a)]$,

$$u_1 = a_1 \operatorname{sh} ky \sin ky + a_2 \operatorname{ch} ky \sin ky + a_3 \operatorname{sh} ky \cos ky + a_4 \operatorname{ch} ky \cos ky - q/2k^2,$$

$$\theta_1 = 1 + a_1 \operatorname{ch} ky \cos ky + a_2 \operatorname{sh} ky \cos ky - a_3 \operatorname{ch} ky \sin ky - a_4 \operatorname{sh} ky \sin ky,$$

where

$$a_1 = -\alpha \operatorname{sh} ky_0 \sin ky_0 + \beta(\operatorname{ch} ky_0 \sin ky_0 + \operatorname{sh} ky_0 \cos ky_0);$$
$$a_2 = \alpha \operatorname{ch} ky_0 \sin ky_0 - \beta(\operatorname{ch} ky_0 \cos ky_0 + \operatorname{sh} ky_0 \sin ky_0);$$
$$a_3 = -\alpha \operatorname{sh} ky_0 \cos ky_0 + \beta(\operatorname{ch} ky_0 \cos ky_0 - \operatorname{sh} ky_0 \sin ky_0);$$
$$a_4 = \alpha \operatorname{ch} ky_0 \cos ky_0 + \beta(\operatorname{ch} ky_0 \sin ky_0 - \operatorname{sh} ky_0 \cos ky_0);$$
$$\alpha = u_2 + q/2k^2;\ \beta = y_0 q/2k.$$

In the special case $q = 0$ we have $y_0 = 1 - \pi/2k$,

$$u_2 = 1/\operatorname{sh}(\pi/2),\ \theta_2 = 1,\ u_1 = \operatorname{ch}(k - ky - \pi/2)\sin(k - ky)/\operatorname{sh}(\pi/2),$$
$$\theta_1 = 1 - \operatorname{sh}(\pi/2 - k + ky)\cos(k - ky)/\operatorname{sh}(\pi/2).$$

It is clear from these expressions that in the center of the channel there is a region whose size depends on k and q where the fluid moves with a velocity constant over the cross section

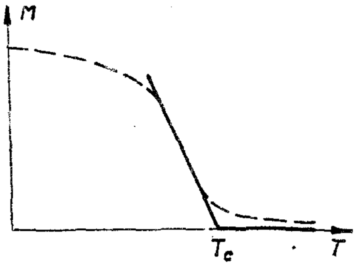

Fig. 1

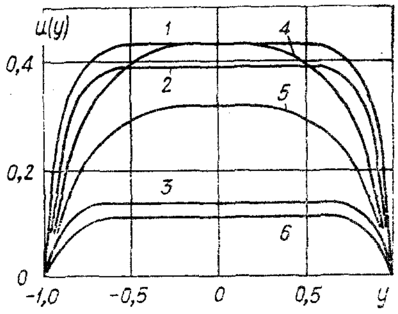

Fig. 2

of the channel. In Fig. 2 we show the dimensionless velocity profile u(y) for different values of k and q (curves 1-3: $k=\pi$, $q=0, 1, 3$; curves 4-6: $k=\pi/2$, $q=0, 1, 3$).

It remains to be shown under what conditions our results will not significantly depend on the form of the approximation function used. In order to answer this question we consider the original boundary value problem in Sobolev space $W_2^{(1)}$. We write (4), (5) and boundary conditions (6) in the form

$$d^2u/dy^2 = -F_1(\theta); \tag{8}$$

$$d^2\theta/dy^2 = -F_2(u, \theta); \tag{9}$$

$$u = \theta = 0 \text{ for } (y = \pm 1), \tag{10}$$

where

$$F_1(\theta) = \begin{cases} 2k^2(1-\theta), & \theta < 1, \\ 0, & \theta \geqslant 1, \end{cases} \quad F_2(u, \theta) = \begin{cases} 2k^2u + q, & \theta < 1, \\ q, & \theta \geqslant 1. \end{cases}$$

Multiply both sides of (8) by u and both sides of (9) by θ, and integrating with respect to y with the use of boundary condition (10) we obtain

$$\int_{-1}^{1}\left(\frac{du}{dy}\right)^2 dy = \int_{-1}^{1} uF_1(\theta)\,dy, \quad \int_{-1}^{1}\left(\frac{d\theta}{dy}\right)^2 dy = \int_{-1}^{1} \theta F_2(u, \theta)\,dy.$$

From the Fridricks inequality [6] we have

$$\int_{-1}^{1} u^2 dy \leqslant c_1 \int_{-1}^{1}\left(\frac{du}{dy}\right)^2 dy,$$

where $c_1 \geqslant 0$ is a constant. It then follows that

$$\|u\|_1^2 \leqslant (1+c_1)\int_{-1}^{1} uF_1(\theta)\,dy$$

and similarly

$$\|\theta\|_1^2 \leqslant (1+c_1)\int_{-1}^{1} \theta F_2(u, \theta)\,dy,$$

where $\|\cdot\|_1$ denotes the norm in the space $W_2^{(1)}$. Thus we obtain

$$\|u\|_1^2 + \|\theta\|_1^2 \leqslant (1+c_1)\left[\int_{-1}^{1} uF_1(\theta)\,dy + \int_{-1}^{1} \theta F_2(u, \theta)\,dy\right].$$

and it then follows that

$$\|u\|_1^2 + \|\theta\|_1^2 \leqslant \frac{1}{2}\left[\int_{-1}^{1} u^2 dy + \int_{-1}^{1} \theta^2 dy\right] + \frac{(1+c_1)^2}{2}\int_{-1}^{1}\left[F_1^2(\theta) + F_2^2(u, \theta)\right] dy.$$

Finally we obtain the required inequality

$$\|u\|_1^2+\|\theta\|_1^2 \leqslant c(\|F_1\|_0^2+\|F_2\|_0^2), \tag{11}$$

where $\|\cdot\|_0$ is the norm in the space L_2. With the help of (11) it can be shown that the solution of problem (8)-(10) is stable with respect to small perturbations of the right-hand sides $-F_1$ and $-F_2$.

We consider the perturbed problem

$$d^2u_\varepsilon/dy^2 = -F_{1\varepsilon}(\theta_\varepsilon),\ d^2\theta_\varepsilon/dy^2 = -F_{2\varepsilon}(u_\varepsilon,\ \theta_\varepsilon),$$

where $F_{i\varepsilon} \to F_i$ when $\varepsilon \to 0$

$$u_\varepsilon = \theta_\varepsilon = 0 \ \text{ for } \ y = \pm 1,$$

and compare with the original problem (8)-(10). Let $\omega = u_\varepsilon - u$ and $\tau = \theta_\varepsilon - \theta$ and consider the problem for the differences

$$d^2\omega/dy^2 = -[F_{1\varepsilon}(\theta_\varepsilon) - F_1(\theta)],\ d^2\tau/dy^2 = -[F_{2\varepsilon}(u_\varepsilon,\theta_\varepsilon) - F_2(u,\theta)],$$
$$\omega = \tau = 0 \ \text{ for } \ y = \pm 1. \tag{12}$$

Applying inequality (11) to (12) we obtain

$$\|\omega\|_1^2+\|\tau\|_1^2 \leqslant c\left[\int_{-1}^{1}(F_{1\varepsilon}-F_1)^2\,dy+\int_{-1}^{1}(F_{2\varepsilon}-F_2)^2\,dy\right],$$

from which it follows that when the parameter k is bounded, the original problem is stable.

Thus we have shown that for flows at temperatures near the Curie point, the characteristic feature of the solution is the formation of a region of fluid (the flow core) moving with a velocity constant over the cross section of the channel. Naturally in the exact solution the fluid velocity will not be strictly constant over the entire cross section of the core.

We computed the velocity profile and temperature by numerical methods on a computer for other more realistic approximation functions to the curve $M = M(T)$. The calculations showed that for bounded values of the parameter k, differences in the velocity profile and temperature were insignificant, and this supports the validity of the treatment given above.

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